Algebra 2, Quarter 2, Unit 2.1
Quadratics and Other Polynomials

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Know and apply the Fundamental Theorem of Algebra and understand that it holds true for quadratics.
- Analyze polynomial functions by examining a graph or a table and find maximums, minimums, intercepts, increasing, decreasing, end behavior, and symmetries.
- Compare and contrast the characteristics of higher degree polynomial functions that are given in different forms (algebraically, graphically, numerically in tables or verbal.)
- Know and apply the Remainder Theorem (using synthetic division to get zeros and factors).
- Graph functions given the equation-identify zeros and end behavior. (Use technology for more complicated cases.)
- Given a polynomial function, rewrite in different forms (standard, factored, and vertex form) to reveal zeros, extreme values, and symmetry of the graph (include word problems).

Mathematical practices to be integrated

- Construct viable arguments and critique the reasoning of others.
- Analyze situations by breaking them into simpler cases.
- Justify conclusions and communicate them to others.
- Determine whether arguments are correct.
- Decide whether conclusions make sense, and ask useful questions to clarify or improve the conclusions.

Model with mathematics.

- Apply the mathematics known to solve problems arising in everyday life, society, and the workplace.
- Describe how one quantity of interest depends on another.
- Make assumptions and approximations to simplify a complicated situation.

Use appropriate tools strategically.

- Uses the available tools (including calculators) when solving a mathematical problem.
- Detect possible errors by using other mathematical knowledge.
- Use technology to validate assumptions.

Essential questions

- How is the Fundamental Theorem of Algebra used to tell us about the x-intercepts and the roots of a function?
- How is the Remainder Theorem used to find the zeros and factors of a polynomial?
• What are the similarities and differences among polynomial functions with varying degrees?
• What is the connection between the graphical and algebraic representation of polynomial functions?
• What are the advantages and disadvantages of using technology to analyze graphs of polynomials?

Written Curriculum

Common Core State Standards for Mathematical Content

The Complex Number System

N-CN

Use complex numbers in polynomial identities and equations. [Polynomials with real coefficients]

N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Interpreting Functions

F-IF

Interpret functions that arise in applications in terms of the context [Emphasize selection of appropriate models]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations [Focus on using key features to guide selection of appropriate type of model function]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Arithmetic with Polynomials and Rational Expressions

Understand the relationship between zeros and factors of polynomials

A-APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5  **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**Clarifying the Standards**

**Prior Learning**

Students have solved problems by computation since grade 2. In grade 4, students demonstrated conceptual understanding of linear relationships and constant rates of change. This continued through grade 6, where the change of the independent variable affecting the dependent variable was introduced. Students in grades 7–9 were introduced to nonlinear relationships and varying rates of change, and students learned the differences between constant and varying rates of change.

Students in grade 9 were required to understand and apply linear and nonlinear relationships, including domain, range, maximum, minimum, intercepts, increasing, and decreasing. In algebra 1, they learned how to factor, identify, and understand the relationship between zeros and factors of polynomials.

**Current Learning**

Students master knowledge of the characteristics of linear functions, including domain, range, maximum, minimum, intercepts, increasing, decreasing, zeros, and end behavior.

**Future Learning**

Students will demonstrate conceptual understanding of one-to-one and onto, including critical points, inflection points, additional extensions of end behavior, odd, even, continuity, and limits to other types of functions. They will also analyze domain restrictions. These topics will expand to rational, exponential, logarithmic, and trigonometric functions.

**Additional Findings**

*Principles and Standards for School Mathematics* speaks to the importance of representing and analyzing mathematical situations and structures. Research indicates that fluency in algebraic concepts improves problem-solving abilities in many other areas (p. 300).
Algebra 2, Quarter 2, Unit 2.2
Application of Polynomials

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned
- Given a quadratic word problem in one variable, create an equation and solve (e.g., \(x^2 + 2x + 7 = 10\), free fall problems, area problems, etc.).
- Given a quadratic word problem in two or more variables, create an equation, and graph. (e.g., projectile motion, pool, picture frame, etc.).
- State limitations/restrictions by writing equations or inequalities in the context of the problem.
- Determine whether or not solutions are viable in the context of the problem.
- Identify the implications to a graph due to limitations based on a problem situation. (See example given with F-IF-5.)
- Examine quadratic-like polynomials and rewrite them with quadratic factors. (See example given with A-SSE.2.)
- Use factorization or factored form to identify zeros and construct a rough graph of the function. (e.g., \(y = -x^2(x-1)(x+2)\) or \(y = x^2 - 3x - 4\))
- Prove that the two sides of a polynomial identity equation are equal by manipulating one or both side of the identity.
- Describe numerical relationships by using the polynomial identities.
- Explain why the x-coordinate of the point of intersection of the system \(y = f(x)\) and \(y = g(x)\) is the solution to \(f(x) = g(x)\).
- Find the approximate solutions of \(f(x) = g(x)\) using technology, including linear and polynomial functions.

Mathematical practices to be integrated
- Reason abstractly and quantitatively.
- Make sense of relationships and problems.
- De-contextualize a given situation.
- Manipulate symbols and problems.
- Use different properties of operations for quantitative reasoning.

Look for and make use of structure.
- Look for patterns.
- Use a simpler problem to break down more complicated problems.
Essential questions

- What is the difference between solving word problems that involve one-variable equations and solving word problems that involve two-variable equations?
- Why are there limitations/restrictions on variables when solving real-life problems?
- Why is it important to be able to manipulate polynomials into different algebraic forms?
- Why is the x-coordinate of the point of intersection of the system $y = f(x)$ and $y = g(x)$ the solution to $f(x) = g(x)$?

Written Curriculum

Common Core State Standards for Mathematical Content

### Arithmetic with Polynomials and Rational Expressions A-APR

Use polynomial identities to solve problems

A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal’s Triangle.\(^1\)

\(^1\) The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

### Creating Equations* A-CED

Create equations that describe numbers or relationships [Equations using all available types of expressions, including simple root functions]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

### Seeing Structure in Expressions A-SSE

Interpret the structure of expressions [Polynomial and rational]

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
**Arithmetic with Polynomials and Rational Expressions**  
**A-APR**

**Use polynomial identities to solve problems**

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A-APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity \((x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\) can be used to generate Pythagorean triples.*

**Interpreting Functions**  
**F-IF**

**Interpret functions that arise in applications in terms of the context** *[Emphasize selection of appropriate models]*

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function \(h(n)\) gives the number of person-hours it takes to assemble \(n\) engines in a factory, then the positive integers would be an appropriate domain for the function.*

**Reasoning with Equations and Inequalities**  
**A-REI**

**Represent and solve equations and inequalities graphically** *[Combine polynomial, rational, radical, absolute value, and exponential functions]*

A-REI.11 Explain why the \(x\)-coordinates of the points where the graphs of the equations \(y = f(x)\) and \(y = g(x)\) intersect are the solutions of the equation \(f(x) = g(x)\); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \(f(x)\) and/or \(g(x)\) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**Common Core Standards for Mathematical Practice**

2 **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Cumberland, Lincoln, and Woonsocket Public Schools, with process support from the Charles A. Dana Center at the University of Texas at Austin
7. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

**Clarifying the Standards**

*Prior Learning*

In grade 8, students solved linear equations and systems of linear equations. Students also constructed functions to model information. In unit 1.3 of algebra 2, students identified zeros and made rough graphs of polynomial functions.

*Current Learning*

In this unit, students are introduced to Pascal’s Triangle and how it relates to expanding a polynomial through the Binomial Theorem. Students continue to expand their problem-solving skills by examining free fall, area, and projectile motion problems and how they relate to quadratic functions and inequalities. Students expand their knowledge of factoring to quadratic-like polynomials. Students work backwards from given factors to create rough sketches of graphs of polynomials using knowledge of zeros and end behavior.

*Future Learning*

In unit 3.3 of algebra 2, students will work with trigonometric identities. Students will study more polynomials and algebra manipulation in precalculus, calculus, and other college courses.

*Additional Findings*

*A Research Companion to Principles and Standards for School Mathematics* discusses functions and solutions on a graph (pp. 132–133).

*Beyond Numeracy* discusses the quadratic formula being the first theorem proved in high school algebra. The book also discusses approximating versus exact roots and links this content to physics problems (pp. 198–199).
Algebra 2, Quarter 2, Unit 2.3

Rational Functions

Overview

Content to be learned

• Add, subtract and multiply rational expressions.
• Divide rational expressions using inspection, long division, and synthetic division for both linear and quadratic denominators.
• Solve rational equations including extraneous solutions.
• Graph rational functions with appropriate labels, scales, intercepts, and horizontal and vertical asymptotes.
• Find the domain, range, vertical asymptotes, and horizontal asymptotes of a rational function by analyzing the graph and the function in factored form.
• Mark increasing and decreasing intervals and determine end behavior.
• Determine the inverse of a rational function.
• Use technology to create a table of values, and graph and approximate solutions of rational functions.

Mathematical practices to be integrated

Model with mathematics.
• Use a function to describe how one quantity depends on another.
• Make assumptions and approximations to simplify complicated situations.
• Identify important quantities using graphs.
• Determine if results make sense.

Attend to precision.
• State the meaning of symbols and give clear definitions for vocabulary.
• Carefully label intercepts and asymptotes, and mark increasing and decreasing intervals.

Essential questions

• How do rational functions differ from polynomial functions?
• Either by looking at the graph or analyzing the equation, how do you know there are discontinuities in a rational function?
• How are critical values used to describe a complete graph of a rational function?
• How can rational functions be described using multiple representations?
• How are a rational function and its inverse related?
Written Curriculum

Common Core State Standards for Mathematical Content

**Creating Equations**

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<th>Common Core State Standard</th>
<th>Description</th>
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<tbody>
<tr>
<td>A-CED.2</td>
<td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*</td>
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<tr>
<td>A-CED.4</td>
<td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance $R$.*</td>
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**Arithmetic with Polynomials and Rational Expressions**

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<tr>
<td>A-APR.6</td>
<td>Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</td>
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**Reasoning with Equations and Inequalities**

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<tr>
<td>A-REI.2</td>
<td>Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</td>
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<tr>
<td>A-REI.11</td>
<td>Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*</td>
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Interpreting Functions

Interpret functions that arise in applications in terms of the context [Emphasize selection of appropriate models]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Building Functions

F-BF.4 Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2x^3 or f(x) = (x+1)/(x-1) for x ≠ 1.

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grades K–3, students were introduced to addition and, in grade 3, multiplication and division. In grades 3–4, students were introduced to fractions as numbers. In grade 5 students added and subtracted fractions. Grade 6 instruction included common factors and multiples and, in grade 7, students added, subtracted, multiplied, and divided rational numbers. In algebra 1, students performed operations on polynomials.

Current Learning

In this unit, students perform operations with rational expressions to include division by inspection, long division, and synthetic division. Students learn that a rational function can be viewed as a quotient of polynomial functions with appropriate domain restrictions in the denominator. Students graph and solve rational functions with and without extraneous solutions. The characteristics of rational functions are developed including domain, range, asymptotes, maximum, minimum, intercepts, increasing, and decreasing. Students are introduced to inverses.

Future Learning

In grade 12, students will continue to expand graphing rational functions.

Additional Findings

Principles and Standards for School Mathematics discusses how to represent and analyze mathematical situations and structures using algebraic symbols. According to this source, fluency with algebraic symbolism helps students represent and solve problems in the curriculum. Students need to be fluent in generating algebraic expressions, combining them and re-expressing them in alternative forms (p. 300).