Precalculus, Quarter 1, Unit 1.1
Conic Sections—Ellipses and Hyperbolas

Overview

Number of instructional days: 12  (1 day = 45–60 minutes)

Content to be learned

- Derive the equations of ellipses and hyperbolas using given information.
- Translate characteristics of ellipses and hyperbolas between geometric and algebraic representations.

Mathematical practices to be integrated

- Reason abstractly and quantitatively.
- Decontextualize by generating equations for ellipses and hyperbolas, given a real-world situation.
- Determine the similarities and differences between the sum distances from the foci of an ellipse/hyperbola to the points involved in the conic section.
- Attend to precision.
- Verify the equations and properties of ellipses and hyperbolas through both geometric and algebraic representation.

Essential questions

- What are the similarities and differences among the four types of curves known as conic sections?
- What is a conic section? How is it developed?
- What is the intersection of a cone and a plane parallel to a line along the side of a cone?
- What mathematical theorems and postulates are used in finding the equations of conic sections?
- What is meant by a locus of points? How is it used in determining an equation of a conic section?
Written Curriculum

Common Core State Standards for Mathematical Content

<table>
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<tr>
<th>Expressing Geometric Properties with Equations</th>
<th>G-GPE</th>
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<tr>
<td>Translate between the geometric description and the equation for a conic section</td>
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<tr>
<td>G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.</td>
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Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Clarifying the Standards

Prior Learning

Students learned how to use variables in expressions and solve one-variable equations in grade 6, and they constructed polygons on a coordinate plane. In grade 7, students solved algebraic equations and drew, constructed, and described geometric figures. In grade 8, students learned integer exponents. In algebra 1, students used quadratic equations to solve problems on and off the coordinate plane. In geometry, students derived the formulas for conics, including circles and parabolas. In algebra 2, students learned how to identify and apply domain and range restrictions when graphing various types of equations. Algebra 2 students also worked with circles in trigonometric situations.

Current Learning

Precalculus students develop equations of ellipses and hyperbolas. They must draw on their experiences in geometry with the equations of circles and parabolas in order to complete their initial look at conic sections in high school.

Future Learning

Calculus students will need to have an in-depth understanding of the equations and graphs of conics. Many careers also require applications of these concepts, including engineering, architectural design (see the Mormon Tabernacle auditorium), and related fields.

Additional Findings

*Beyond Numeracy* states that analytical geometry and its offshoots are seemingly natural and thus taken for granted, that it sometimes requires a special effort to remember that they are inventions of human beings. By examining graphs of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, general quadratic equations in two variables give rise to equations whose graphs are circles, ellipses, parabolas, and hyperbolas. These are the same figures that are formed by the intersection of a cone and plane, where the angle of the plane determines which one of the conic sections results (pp. 11–14, 199–200).
Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

• Graph rational functions to ascertain their key characteristics.
• Analyze rational function graphs for use in real-world situations.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.
• Analyze givens, constraints, relationships, and goals.

Use appropriate tools strategically.
• Graph rational functions using available technology, including graphing calculators and spreadsheet programs.
• Demonstrate how to graph rational functions using technology.
• Use viewing windows to find zeros and asymptotes.

Model with mathematics.
• Use the graph of a rational function to describe a problem situation.

Essential questions

• What are the identifying characteristics of graphs of rational functions?
• Where are rational functions used in real-world situations?
• Why is it important to use graphing calculator technology when studying rational functions?
• How does the graph of a rational function display the characteristic of end behavior?
• What does a hole in a rational function graph depict?
• How are a rational function and its inverse related?
Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions

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<td>F-IF.7</td>
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</tr>
<tr>
<td>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</td>
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</tr>
</tbody>
</table>

Common Core Standards for Mathematical Practice

1  Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5  Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
Clarifying the Standards

Prior Learning

In grade 6, students continued their previous work with whole numbers to develop an understanding of rational numbers. They also learned how to relate graphs and tables to equations. In grade 7, these studies continued with operations with fractions and integers. In grade 8, students worked with equations and graphs and began the study of linear and nonlinear functions. In algebra 1, the focus was on linear and exponential functions. Algebra 1 students also compared two functions algebraically (i.e., compare two linear functions or two exponential functions). In algebra 2, students focused on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. They worked with polynomial, quadratic, simple rational and square root, trigonometric, cube root, and piecewise-defined functions, including step functions and absolute value functions. Students graphed exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. They also used transformations of functions on all functions studied in algebra 2.

Current Learning

Precalculus students are responsible for the reinforcement and mastery of graphing rational functions. They must be able to identify the zeros and asymptotes and explain the significance of the end behavior of the function. They begin to use these types of functions in real-life situations and applications.

Future Learning

This content will be used in all subsequent mathematics courses. Rational functions are an integral part of college algebra, where students must manipulate rational equations. Knowledge of rational functions will be imperative in calculus and in bridge building. Rational functions will also be used in engineering applications.

Additional Findings

NCTM’s Principles and Standards for School Mathematics discusses how to represent and analyze mathematical situations and structures using algebraic symbols. According to this source, fluency with algebraic symbolism helps students represent and solve problems in the curriculum. Students need to be fluent in generating algebraic expressions, combining them and re-expressing them in alternative forms (p. 300).
Precalculus, Quarter 1, Unit 1.3
Inverse Functions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned
- Solve an equation of the form \( f(x) = c \) for a simple function \( f \) that has an inverse \( f^{-1}(x) \).
- Find the inverse \( f^{-1}(x) \) of a simple function \( f \) that is of the form \( f(x) = c \).
- Show that two functions \( f(x) \) and \( g(x) \) are inverses of each other through the use of composition.
- Read and interpret values of a function and its inverse through the use of graphs and tables.
- Create an inverse function for a non-invertible function by restricting the domain.

Mathematical practices to be integrated
- Make sense of problems and persevere in solving them.
- Use multiple representations (graph, table, equation, verbal description).
- Work between different representations.
- Justify the relationship between a function and its inverse.
- Construct viable arguments and critique the reasoning of others.
- Recognize the differences between invertible and non-invertible functions.
- Identify the need for restrictions of domain of a function to find an inverse function.

Essential questions
- How are a function and its inverse related?
- How can one show using various representations that a function has an inverse?
- What are the similarities and differences between a function, its reciprocal, and its inverse?
- Why does restricting the domain of a function allow you to find an inverse of a noninvertible function?
- What are some real-life applications in which one has to restrict the domain of a function so that its inverse is a function?
Written Curriculum

Common Core State Standards for Mathematical Content

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**Build new functions from existing functions**

F-BF.4    Find inverse functions.

b.  (+) Verify by composition that one function is the inverse of another.

c.  (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d.  (+) Produce an invertible function from a non-invertible function by restricting the domain.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Clarifying the Standards

Prior Learning

Students have analyzed functions and their properties using different representations. In algebra 1, students were introduced to the function definition and the use of the f(x) function notation. The focus in algebra 1 was on linear, quadratic and exponential functions, and arithmetic and geometric sequences. In algebra 2, students learned to identify when functions are undefined and graphed intercepts, intervals where functions are increasing and decreasing, positive or negative symmetries, and end behavior. In addition the study of inverse functions in an applied context was developed.

Current Learning

Students work with function notation, f⁻¹(x), and properties of functions in various applied contexts. The analysis of functions is undertaken using multiple representations (graphs, tables, equations, verbally, sequentially, and pictorially).

Future Learning

Students will continue to extend their knowledge of functions and their inverses to include logarithmic, exponential, and trigonometric Functions. Domain restrictions will continue to play a pivotal role in the development of inverse functions. In calculus, the derivatives and integrals of functions and their inverses will be analyzed in depth.

Additional Findings

Principles and Standards for School Mathematics speaks to the importance of representing and analyzing mathematical situations and structures. Research indicates that fluency in algebraic concepts lend improves problem-solving abilities in many other areas (p. 300).

Science for All Americans discusses models that can be used to relate to equations (pp. 168–172).

Precalculus, Quarter 1, Unit 1.4
Composite Functions

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned
- Create and describe compositions of functions both verbally and algebraically.

Mathematical practices to be integrated
- Model with mathematics.
  - Describe a real-life situation using composition of functions.
  - Express and analyze composition of functions algebraically, verbally, pictorially, and graphically.
- Attend to precision.
  - Communicate precisely what the composite function notation is.

Essential questions
- What are real-life situations in which the composition of functions can be applied?
- What is a composition function?
- What basic operations and properties can be applied within a composition of functions?
- How does composing two continuous functions affect the continuity of the new function?
# Written Curriculum

## Common Core State Standards for Mathematical Content

<table>
<thead>
<tr>
<th>Building Functions</th>
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<tbody>
<tr>
<td>Build a function that models a relationship between two quantities</td>
<td></td>
</tr>
<tr>
<td>F-BF.1 Write a function that describes a relationship between two quantities.</td>
<td></td>
</tr>
<tr>
<td>c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</td>
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</table>

## Common Core Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**

   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

6. **Attend to precision.**

   Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
Clarifying the Standards

Prior Learning

In grade 6, students learned to represent and analyze quantitative relationships between independent and dependent variables. In grade 8, linear functions were the primary focus. In algebra 1, students continued to explore applications of exponents—they explored functions and function notation and analyzed linear, exponential, and quadratic functions. They also built functions that model relationships between two quantities. Algebra 2 students analyzed various types of functions, including comparing properties of two functions and introducing inverse functions.

Current Learning

Precalculus students are responsible for the development, review, and mastery of composition of functions. These concepts are important for their understanding of inverse functions and operations with functions.

Future Learning

Composition of functions will be used extensively in upper level mathematics courses and in business and industry. Many businesses depend on the joining of functions to produce a mathematical model indicative of fiscal growth in their companies. Composition of functions also has various applications in engineering fields. The concept of composition of functions is crucial to the understanding of continuity in calculus.

Additional Findings

Principles and Standards for School Mathematics states that students must know what it means to compose functions, including the role of the inner and outer functions and the numbers on which they act in the composition (p. 302).

Principles and Standards for School Mathematics speaks to the importance of representing and analyzing mathematical situations and structures. Research indicates that fluency in algebraic concepts improves problem-solving abilities in many other areas (p. 300).